

1 Scale dependence of the hydraulic properties of a
2 fractured aquifer estimated using transfer functions

D. Pedretti, Dept. of Earth, Ocean and Atmospheric Sciences, University of British Columbia,
2207 Main Mall, Vancouver, British Columbia, Canada V6T1Z4 (dpedretti@eos.ubc.ca)

A. Russian, Geosciences Montpellier, Universite' de Montpellier, Montpellier Cedex05, France

X. Sanchez-Vila, Hydrogeology Group (GHS, UPC-CSIC), Dept. Geotechnical Engineering
and Geosciences, Universitat Politecnica de Catalunya - UPC, Barcelona, Spain

M. Dentz, Hydrogeology Group (GHS, UPC-CSIC), Institute of Environmental Assessment
and Water Research, Spanish National Research Council (IDAEA-CSIC), Barcelona, Spain

3 **Abstract.** We present an investigation of the scale dependence of hydraulic
4 parameters in fractured media based on the concept of transfer functions (TF).
5 TF methods provide an inexpensive way to perform aquifer parameter es-
6 timation, as they relate the fluctuations of an observation time series (hy-
7 draulic head fluctuations) to an input function (aquifer recharge) in frequency
8 domain. Fractured media are specially sensitive to this approach as hydraulic
9 parameters are strongly scale dependent, involving non-stationary statisti-
10 cal distributions. Our study is based on an extensive data set, involving up
11 to 130 measurement points with periodic head measurements that in some
12 cases extend for more than 30 years. For each point, we use a single-porosity
13 and dual-continuum TF formulation to obtain a distribution of transmissiv-
14 ities and storativities in both mobile and immobile domains. Single-porosity
15 TF estimates are compared with data obtained from the interpretation of
16 over 60 hydraulic tests (slug and pumping tests). Results show that the TF
17 is able to estimate the scale dependence of the hydraulic parameters, and
18 it is consistent with the behavior of estimates from traditional hydraulic tests.
19 In addition, the TF approach seems to provide an estimation of the system
20 variance and the extension of the ergodic behavior of the aquifer (estimated
21 in approximately 500 m in the analyzed aquifer). The scale dependence of
22 transmissivity seems to be independent from the adopted formulation (sin-
23 gle or dual-continuum), while storativity is more sensitive to the presence
24 of multiple continua.

1. Introduction

25 Fractured aquifer systems have been receiving increasing attention in the recent years
26 due to their strategic importance for drinking water supply and as a resource for agri-
27 culture and industrial activities. Correct hydraulic parametrization of fractured aquifers
28 requires an integrated approach capable of effectively describing the impact of randomly
29 distributed fractures and matrix hydraulic properties upon the temporally varying flow
30 patterns described at different observation scales. A general review of these concepts,
31 including characterization methods and modeling solutions for fractured media can be
32 found for instance in Berkowitz [2002].

33 Hydraulic parameters, such as aquifer transmissivity (T) and storativity (S), are com-
34 monly estimated by model fitting of observed groundwater fluctuations associated with
35 one or more external stresses (such as natural aquifer recharge or pumping). While most
36 traditional estimation methods rely either on classical model curve fitting [e.g. Zech et al.,
37 2015] or else on inverse calibration approaches [e.g., Zhou et al., 2014], recent applications
38 have focused on transfer function (TF) estimation methods as a potential alternative
39 method [e.g., Denic-Jukic and Jukic, 2003; Liao et al., 2014; Pinault et al., 2001; Trincherro
40 et al., 2011; Jimenez-Martinez et al., 2013]. In TF methods, the aquifer is seen as an ef-
41 fective filter that transforms recharge signals into aquifer head or discharge fluctuations.
42 From the initial formulations of TF methods [Gelhar, 1974], several alternative models
43 based on stationary and non-stationary aquifer assumptions have blossomed [e.g., Zhang
44 and Schilling, 2004; Schilling and Zhang, 2012]. As TF models are usually formulated
45 in the frequency domain, they become particularly suited for the analysis of fractured

46 media, where the hydraulic properties are conveniently represented using non-stationary
47 statistical distributions [e.g., Barton and Larsen, 1985; Bonnet et al., 2001; Berkowitz,
48 2002; Zhang and Li, 2005; Little and Bloomfield, 2010].

49 Fracture media as well as porous media display scale effect of estimated hydraulic pa-
50 rameters [e.g., Brace, 1980, 1984; Clauser, 1992]. This effect occurs since model outputs
51 are sensitive to the support volume of the observations, the support scale of measure-
52 ments and the adopted interpretation method [e.g. Guimerá et al., 1995; Sanchez-Vila
53 et al., 1996; Beckie, 1996; Guimerá and Carrera, 2000; Schulze-Makuch and Malik, 2000;
54 Lai and Ren, 2007]. For instance, in systems characterized by randomly-distributed high-
55 permeable fractures embedded into a low permeable matrix, there is a positive correlation
56 between estimated T and the support scale of hydraulic tests [e.g., Le Borgne et al., 2006].
57 This occurs since a large support scale generally corresponds to a larger probability of
58 sampling high-conductive connected fractures, such that the average T increases with
59 scale. When the support scale is of the order of or larger than the characteristic het-
60 erogeneity scale, estimated T values reach an asymptotic value [e.g., Sanchez-Vila et al.,
61 1996], which defines the scale of aquifer ergodicity. Scale dependence of S has been also
62 reported in the literature and directly linked with aquifer heterogeneity and connectivity
63 as well as the interpretation method used in the hydraulic test data analysis [e.g., Meier
64 et al., 1998; Sanchez Vila et al., 1999].

65 Jimenez-Martinez et al. [2013] discuss the apparent scaling effects of T and S in a het-
66 erogeneous fractured aquifer in Ploemeur (Brittany, France). They compared estimations
67 obtained from traditional hydraulic tests against those obtained from hydraulic responses
68 analyzed by two single-porosity TF models, namely the Linear Model and an approx-

69 imated Dupuit Model [Gelhar, 1974], both of which ignore the spatial dependency of
70 observations related to the distance from the aquifer discharge point (x_L). Compared to
71 traditional hydraulic tests, these authors found that the TF-based approaches provided,
72 on average, larger T and S estimates, combined with low estimation variances, with a con-
73 vergence of T at large scale towards the largest T estimates measured at smaller scales.
74 Moreover, Jimenez-Martinez et al. [2013] obtained S estimates much larger than typical
75 values associated with confined fractured aquifer. The authors explained this observation
76 by indicating that the methods with low support volume (flowmeter and pumping tests)
77 tend to preferentially capture low storativity features, which respond faster to hydraulic
78 perturbations, while TF methods quantify processes at the basin scale, which may have
79 a large overall storage.

80 The objective of this work is to provide insights on scale effects observed in the well-
81 characterized fractured aquifer at the El Cabril Site, located in Southern Spain (Fig. 1a),
82 using single and multi-continuum Dupuit Model (DM) formulations. We specifically focus
83 on x_L as a key parameter to understand scale effects of estimated parameters obtained
84 from TF models. The experimental database used in this work consists of more than 60
85 estimates of hydraulic properties obtained from model fitting of slug and pumping tests
86 performed in several boreholes, sparsely located in the aquifer, and more than 130 head
87 fluctuation time series collected during more than two decades in an equivalent number
88 of boreholes.

89 The first goal of this analysis is to compare estimates of hydraulic parameters obtained
90 from slug and pumping tests against those obtained from the scale-dependent, single-
91 porosity Dupuit Model of Gelhar [1974]. The objective is to evaluate if the scale effects

92 upon estimated T and S values may be directly related to x_L , and therefore to analyze if
93 TF methods are sensitive to the measurement and support scales, in a similar fashion as
94 traditional testing approaches.

95 A second goal is to apply and discuss the results obtained by model fitting of a non-local
96 dual continuum TF formulation derived from the DM solution as reported in Russian
97 et al. [2013]. Several investigations have shown that the anomalous behavior of flow and
98 solute transport in fractured aquifers is sometimes better described and modelled using
99 multicontinuum formulations [e.g. Moench, 1995; Haggerty and Gorelick, 1995; McKenna
100 et al., 2001]. Evidences of effective dual-porosity behavior of El Cabril aquifer were al-
101 ready observed by Sanchez-Vila and Carrera [2004], indicating that the system can be
102 conceptualized as a medium that is composed of an effective fast-flow region (represent-
103 ing the fractures) overlapped to one or multiple low-permeability regions (representing
104 the matrix), all regions exchanging water driven by head gradients. Emphasis is placed
105 in this study toward the sensitivity of the solution to the different parameters involved in
106 the conceptual model.

107 Initially, we introduce the study area in Section 2, focusing on the key geological and
108 hydrogeological aspects and in particular on the measured fracture index. In Section 3
109 we present the estimates of transmissivity and storativity obtained through classical slug
110 and pumping tests and in Section 4 we focus on TF methods. Section 4 includes an
111 introduction of the theoretical single and dual porosity models, the derivation of the
112 experimental TF and an illustrative example. The analysis and the discussion of the
113 results are provided in Section 5 and the conclusion in Section 6.

2. Site Description

2.1. Key Geological and Hydrogeological Aspects

114 El Cabril Site is located in Southern Spain, Fig. 1a, and hosts the Spanish repository for
115 nuclear waste material of low and medium level activity. The fractured aquifer underneath
116 the facility has been widely investigated in the past using multiple approaches, aiming to
117 estimate effective hydraulic and transport properties to become an input in risk assessment
118 exercises [e.g., BRGM, 1990; Carrera et al., 1993; Sanchez-Vila and Carrera, 1997; Meier
119 et al., 1998; Sanchez-Vila and Carrera, 2004; Trincherro et al., 2008].

120 The geological nature of El Cabril aquifer is metamorphic. The main lithologies are
121 biotitic gneisses and metaarkoses, which originated from sedimentary deposits and mag-
122 matic rocks. These materials suffered from several regional structural processes (including
123 high-energy compressive Hercynian deformation), and more recent low-energy localized
124 events. The combination of events resulted in tilting, faulting and an intense net of frac-
125 tures, visible from several outcrops in the area (Fig. 1b). The main orientation of the
126 tilted structures is NW-SE, with fracture planes and sedimentary layers tilted up to 90
127 degrees and directions 60°N to 90°N. A representative geological cross-section, oriented
128 perpendicular to the main direction of faulting, is illustrated in Fig. 2a, showing the
129 different formations, defined by geological criteria and genetic content of the local rocks.
130 The main ones are called Fm *Cabril* (C), Fm *Cuarcitas* (Q), a formation composed of
131 quartz and feldspar with gneisses (QFg), and Fm *Albarrana* (A).

132 Drilling cores from local boreholes and outcrops suggest that fracture spacing is very
133 broad, ranging from 10⁻³m to 10⁻¹m (Fig. 1b). Drilling cores exist for most of the
134 boreholes. An example of these cores is shown in Fig. 1c, where the lengths of intact (i.e.

135 unfractured) core portions are well identified and used to compute the index of fracture
136 intensity (Rock Quality Designation, RQD), a relevant parameter in the analysis and
137 addressed in the next section.

138 The presence of oriented fractures and the topography of the site control the average
139 groundwater dynamics at the site. The morphology of the site is highly irregular, and
140 the hydrological and hydrogeological patterns show a well-defined recharge zone located
141 at high elevation and multiple local discharging locations at topographical lows (Fig.
142 1d). Two minor streams (Morales and Arroyo 4) and a major stream (Montesina) are
143 considered the major discharge feature at the subregional scale, as conceptually depicted
144 in Fig. 1e. Most of the boreholes analyzed in this work are located in the central valley
145 and the crest of the intermediate groundwater divides.

146 The majority of the boreholes existing in the area were used as single-level piezometers
147 to monitor the groundwater fluctuation. Of these, 138 boreholes were constantly moni-
148 tored for several decades, reaching in some cases 30 years. The sampling interval for the
149 majority of the wells was either about 1 day or 15-30 days (*Supplementary Material*),
150 depending on the location. The depth of the piezometers varied between 10s to 100s of
151 meters. A subset of these boreholes were also used to perform several hydraulic and tracer
152 tests. From their analysis, it was concluded that the fracture orientation and intensity
153 generate a strong anisotropy in aquifer hydraulic conductivity, with a major control on
154 groundwater flow patterns. This was corroborated from the analysis of pumping tests
155 and breakthrough curves (BTCs) measured during convergent flow tracer tests performed
156 with different tracers and injecting at different locations in the aquifer, which suggested
157 marked differences in responses displaying strong anisotropic effects [Sanchez-Vila and

158 Carrera, 1997] and fracture connectivity [Trincherro et al., 2008]. Evidence of an effective
159 dual-porosity hydraulic behavior of the aquifer can be inferred from the work of Sanchez-
160 Vila and Carrera [2004], who illustrated that a nonlocal advection-dispersion formulation
161 accounting for fracture-matrix mass exchange was able to fit observed BTCs during the
162 tracer tests, while a single-porosity solution failed to reproduce similar observations.

2.2. Fracture index

As discussed for instance by Jimenez-Martinez et al. [2013], the degree of fracturing of an aquifer can condition the flow patterns and eventually propagate to the estimated parameters. The quality and the integrity of the rock removed from the borehole can be described by the Rock Quality Designation (RQD) Index, which measure the degree of fracturing of the core. RQD is defined from the proportion of the core with intact length larger than 0.1 m [Deere, 1963; Priest and Hudson, 1976]. To calculate this index, intact lengths from drilling boxes are summed up and expressed as a percentage of the total borehole length (B_L), as

$$\text{RQD} = \frac{100}{B_L} \sum_{i=1}^n z_i \quad (1)$$

163 where z_i is the length of the i -th rock fragment exceeding 0.1 m and n is the number of
164 samples ≥ 0.1 m. The larger the RQD, the more intact (i.e., less fractured) the borehole
165 log. Thus, RQD=100% indicates an intact core (no significant fracturing observed). On
166 the other limit, an RQD=0 indicates the core is fully fractured into small pieces.

167 The vertical distribution of RQD in the aquifer was available from a few stratigraphic
168 logs of boreholes reported during drilling operations. Some of these boreholes were also
169 used later for hydraulic testing, allowing for performing a comparison between local degree

170 of fracturing degree and estimated model parameters at different scales (those represen-
171 tative of the tests).

172 We analyzed the original stratigraphic logs of 76 boreholes (distributed all throughout
173 the site), for a total length of approximately 4000 m of borehole scanlines. In each
174 log, RQD was graphically reported alongside the corresponding stratigraphic column.
175 An example is displayed in Fig. 2a, showing three representative stratigraphic columns
176 obtained from the dataset. The black bars beside the vertical geological columns represent
177 the frequency of fracture intensity with depth. The statistics of RQD values used in this
178 work were inferred from the size of these bars (existing data is only graphical). Note that
179 in this figure the vertical scale of the columns is not consistent with the actual lengths of
180 the boreholes but was adapted here for illustrative purposes. Their real vertical size of the
181 columns is reported by the dotted line on top of the geological sketch, which also illustrates
182 the position of the three columns in the aquifer. Fig. 2a illustrates a few important aspects
183 regarding the distribution of inferred fractures in the aquifer, and provides a general idea
184 about the quality and limitation of the available information obtained from our dataset.

185 The first borehole log analyzed (bh1) is shorter than the other two, and explores only
186 the upper part of Fm C. Despite the presence of an upper recent alluvial material, the
187 RQD reported was constant for the entire column, roughly corresponding to RQD=40%.
188 The second point (bh2) spans 40-50m and is characterized by an initial low RQD zone
189 (associated with alluvial deposits), followed by a region with RQD=75% and a subsequent
190 region with RQD=100%, again followed by a final zone with RQD=75%. From the ge-
191 ological sketch, the area of RQD=100% roughly corresponds to fm Q, while the portion
192 with values of 75% are mostly associated to Fm C. In point bh3, the borehole still crosses

193 multiple geological formations; however, RQD seems almost homogeneously distributed
194 along the depth, with an average of 55%-60% independent of the specific formation. An
195 exception is found in an intermediate location where an elevated fracture intensity occurs
196 with RQD=0%. It is emphasized however that, on average, the majority of stratigraphic
197 logs are more similar to bh2 than to bh1 and bh3, which seems to suggest that in most
198 parts of the aquifer the vertical distribution of RQD is heterogeneous and characterized
199 by a sequence of high and low fracturing zones. The importance of this aspect will be
200 clarified later, during the analysis of the estimated hydraulic parameters.

201 The statistical distribution of RQD from all 76 analyzed boreholes is reported in Fig. 2b,
202 in the form of frequency histograms (left) and cumulative density functions (cdfs) (right).
203 It was found that the majority of the borehole logs analyzed display values of RQD>50%,
204 with highest frequency values located in the range 80-90%. However, about 35% of the
205 total explored borehole scanlines show RQD<50%. The red and blue lines indicate the
206 subset of these boreholes which were used to perform slug and pumping tests in the
207 1990s (described below). It is noted here that the statistical distribution of RQD for the
208 boreholes where slug tests were performed provide similar distribution as compared to
209 the full population. This is not the case for the boreholes used for pumping tests where
210 the distribution is shifted towards the left (low RQD values), indicating a bias towards
211 highly-fractured zones in the development of pumping tests.

212 The analysis of the three representative boreholes and the statistics of RQD at the scale
213 of the catchment suggest some important geological aspects of this site. El Cabril aquifer
214 does not systematically present a trend in fracturing index with depth, as observed at
215 the aquifer investigated by Jimenez-Martinez et al. [2013]. RQD varies with depth in an

216 unstructured random way. The comparison between bh2 and bh3 in Fig. 2 suggests in
217 addition that there is no clear correlation between RQD and the type of formations within
218 this aquifer. This is consistent with the presence of post-depositional tectonic effects of
219 the site, affecting all geological formations regardless of their genetic origin.

220 At the scale of the catchment, RQD is negatively skewed. The majority of the aquifer
221 presents very few fractures and a generally intact (i.e., low fractured) matrix. This result
222 is in line with past analyses made on this site and agrees with the general conclusions
223 made on the regional hydraulic behavior of this aquifer, which is expected to behave as
224 a low-permeable crystalline formation, in which a few highly conductive features carry
225 the majority of water. This result is also consistent with common observations made
226 on rock apertures [e.g., Tsang and Tsang, 1989]. Intuitively, one might expect that
227 the aquifer permeability could be inversely correlated to RQD (i.e., permeability increas-
228 ing with fracture intensity). Therefore, the permeability distribution at the scale of the
229 catchment may be expected as positively skewed, with a larger amount of low-permeable
230 zones and fewer high-permeable zones, consistent with typical observations on rock for-
231 mations [e.g., Gustafson and Fransson, 2005]. It is finally noted that aquifer permeability
232 may not directly correlate with 'static' indicators, such as RQD. Indeed, hydraulic prop-
233 erties are effective dynamic parameters and therefore require a dynamic solution to be
234 properly estimated [e.g., Le Borgne et al., 2006]. This is an essential point of this discus-
235 sion, since it motivates the presence of scaling effects of estimated hydraulic parameters.
236 The application of theoretical approaches that quantitatively relate RQD or similar ge-
237 omechanical fracture indexes with rocks permeability [e.g., Liu et al., 1999] need to be

238 inspected carefully before being applied to estimate regional hydrodynamic behavior of
239 fractured aquifers.

3. Hydraulic Parametrization from Slug and Pumping Tests

240 Several hydraulic tests were performed from the 1990s on to characterize the behavior
241 of El Cabril using a suite of techniques. We focus here only on slug tests and pumping
242 tests. In the former, head change is recorded as a function of time at the same well where
243 an instantaneous stress is applied, while in the latter, the head change is observed both at
244 the active well where continuous pumping is performed, and at a number of piezometers
245 located nearby. The different support scale between the two types of tests (larger from
246 pumping tests than for slug tests) results in scaling effects of estimated T and S , as
247 reported by Meier et al. [1998].

248 Slug tests were interpreted under the assumption of 2-D radial flow in a homogeneous,
249 single porosity aquifer using the method of Papadopoulos et al. [1973]. The tests were
250 performed in locations distributed across three of the different geological formations (Fms
251 C, Qfg, and A). The results for estimated T and S in terms of cumulative frequencies
252 from the existing 18 slug tests are reported in Fig. 3. This same subset of boreholes was
253 the one used to construct the histogram corresponding to the RQD index (Fig. 2b).

254 Two long-term pumping tests were performed in the early 1990s [e.g., BRGM, 1990].
255 The first one was made around pumping well S33. The drawdown curves from nine
256 observation boreholes located nearby were interpreted for estimated T and S using the
257 code MARIAJ [Carbonell and Carrera, 1992], which is based on a single-porosity solution.
258 The original reports indicated that S33 was all drilled in Fm C, mainly composed by
259 gneisses. No stratigraphic log (or RQD values) are available for S33. The piezometers

260 were located into two different formations (Qfg and Q). According to field observations,
261 piezometers drilled in Fm Q had a faster and larger response than those in Fm Qfg. A
262 second long-term test implied pumping at well S401, located far from S33 but also drilled
263 in Fm C. Drawdown curves from seven piezometers located in the proximity of the well
264 were monitored and used to estimate T and S using the same methodology as for the
265 previous test. Boreholes were drilled in three different formations (Fms C, Q, QFg).

266 Several short-term pumping tests were also performed in other boreholes located all
267 throughout the site. A similar single-porosity modeling approach was used for the in-
268 terpretation, although no specific details regarding the geological formations explored by
269 these tests were available. In total, the number of estimated parameters from short-term
270 and long-term pumping tests was 42 estimates of T and 30 estimates of S . These results
271 are plotted in Fig. 3 in the form of cumulative frequencies.

272 Comparing estimated values from slug and pumping tests in Fig. 3, it can be observed
273 that the estimated values display the typical scaling effect associated with the different
274 support scales for the corresponding hydraulic tests. Estimates from slug tests show lower
275 average T and S values and a higher variability than those coming from pumping tests.
276 T estimates range over 4 orders of magnitude for slug tests, being around 2 orders for
277 pumping tests. Regarding the estimated S values, the variability ranges over more than 5
278 orders of magnitude for slug tests and about 3 for pumping tests. It is interesting to note
279 that the resulting estimates of S display a range of values spanning from typical values
280 for confined aquifers ($S \in [10^{-4}, 10^{-5}]$) to values representative of unconfined aquifers
281 ($S \approx 10^{-1}$).

282 Estimated T and S values reveal that the aquifer is not only highly heterogeneous, but
283 also characterized by a different effective pressure status depending on the test locations.
284 The pressure status depends directly on the number of confined/unconfined units and
285 potentially by the fracture intensity from the boreholes where these tests were performed.
286 Fig. 2b shows that the cumulative frequency of RQD for the boreholes used in the
287 pumping tests (red lines/bars) is significantly different than that from slug tests (blue
288 lines/bars). Specifically, RQD corresponding to the former display a larger amount of low
289 RQD values as compared to that corresponding to slug tests. In particular, no boreholes
290 with $RQD > 90\%$ were reported in the subset corresponding to locations where pumping
291 tests were performed.

292 The statistical difference in RQD between both populations may contribute to explain
293 the differences observed in Fig. 3, as well as the associated scaling effects between the esti-
294 mates for the two test types. The effective support scale of each hydraulic test depends on
295 the amount of heterogeneity which is sampled by the specific test. Hence, hydraulic tests
296 performed in low-RQD zones result in relatively larger T estimates with lower variability
297 than tests performed in areas with higher RQD. Low RQD values mean short lateral con-
298 tinuity of fractures, which can be associated with a lower hydraulic connectivity of the
299 system and quasi homogeneous hydraulic properties around each borehole. Contrarily, a
300 high fracture intensity can determine a vertical continuity between the ground surface and
301 the subsurface. This may explain why larger estimated S values are reported for pumping
302 tests as compared to those for slug tests.

4. Parameter Estimation using Transfer-Function-Based Methods

303 The transfer function is generally conveniently defined in frequency domain as the ration
 304 of the power of the spectrum of the aquifer response (e.g. hydraulic head fluctuation at a
 305 piezometer) to an input signal (e.g. recharge, r [m]), such as

$$\text{TF} = \left| \frac{h(x, \omega)}{r(\omega)} \right|^2, \quad (2)$$

306 where $h(x, \omega)$ is the hydraulic head [m] for given position x [m] and frequency ω [1/d].
 307 Parameter estimation using TF approaches is based on model fitting of closed-form ana-
 308 lytical TF solutions to match experimental TFs. Estimated parameters directly depend
 309 on the selected formulation and the type of boundary conditions (BCs) applied at the
 310 outflow boundary of the aquifer, and the type of flow formulation (e.g. single or multi-
 311 porosity domain). The reader is referred to Russian et al. [2013] for an exhaustive review
 312 of these concepts. In the following, we present the transfer functions for the single and
 313 dual domain Dupuit models, which we adopted for the analysis of the El Cabril site.

4.1. Dupuit Model (DM)

314 The first model adopted in this work is the single-porosity Dupuit model (DM) by Gelhar
 315 [1974]. The DM describes flow in the aquifer based on the linearized Dupuit-Forchheimer
 316 model [e.g., Bear, 1972]. The model takes the form of

$$S \frac{\partial h(x, t)}{\partial t} = T \frac{\partial^2 h(x, t)}{\partial x^2} + r'(t), \quad (3)$$

317 where S is the storage coefficient [-], t is time [d], T is the transmissivity [m^2/d] and $r'(t)$
 318 is the aquifer recharge rate per unit surface [m/d], which is assumed to be homogeneous.
 319 For the initial condition ($h_0 = 0$) and a Dirichlet BC at the outfall, the TF reads as
 320 [Russian et al., 2013]

$$\text{TF}_{\text{DM}} = \frac{1}{\omega^2 S^2} \left| 1 - \frac{\cosh [\sqrt{i\omega\tau} (1 - x_L/L)]}{\cosh (\sqrt{i\omega\tau})} \right|^2 \quad (4)$$

321 where $i = \sqrt{-1}$ is the imaginary unit and $\tau = L^2 S/T$ is the aquifer response time [d].

322 The inverse of the aquifer response time is named aquifer response rate (ω_L) [Erskine
323 and Papaioannou, 1997] and defines one characteristic frequency of the model, such as
324 $\omega_L = \tau^{-1}$.

325 As pointed out in Russian et al. [2013], x_L identifies another characteristic frequency
326 given by the mean diffusion time from the observation point to the discharge point, such
327 as $\omega_x = T/(x_L^2 S)$. These two characteristic frequencies determine the scaling behaviour
328 of the TF:

- 329 • for $\omega \ll \omega_L$ the TF is flat, which means that long-time components in the recharge
330 spectrum, with frequency lower than the aquifer response rate are not smoothed by the
331 aquifer;
- 332 • for $\omega \gg \omega_x$ the characteristic scaling of TF for the DM is $\text{TF}_{\text{DM}} \propto \omega^{-2}$;
- 333 • for $\omega_L \ll \omega \ll \omega_x$ and if $x_L \ll L$, a third regime develops, where $\text{TF}_{\text{DM}} \propto \omega^{-1}$.

334 We refer the reader to Russian et al. [2013] for details. The distance from the domain
335 boundaries, here from the discharge boundary defines the sampling scale of aquifer het-
336 erogeneity that influences the hydraulic head response to the recharge signal. For a well
337 located close to the outfall or the watershed, the sampled heterogeneity scales are of the
338 order of or smaller than the distance to the respective boundary.

4.2. Dual Continuum Non-local Dupuit Model (DC)

339 The second model considered is the dual-continuum (DC) non-local TF model devel-
340 oped in Russian et al. [2013]. The model mimics the presence of non-equilibrium effects
341 associated with water storage in multiple low-permeable zones within the aquifer using
342 an effective formulation and allows for a broader range of possible scalings of the TF with
343 frequency. The selection of the DC model is based on dual continuum behaviors observed
344 for tracer tests at the El Cabril site.

345 In the dual continuum approach, the aquifer is conceptually represented by two zones
346 or "domains": a mobile domain (m), representing the fractures, and an immobile (im)
347 domain, representing the matrix. Water moves mainly through the highly conductive
348 fractures according to the hydraulic gradient, and may be transferred into the matrix
349 where it is stored for a certain time. The transfer rates between the mobile and immobile
350 domains are encoded in a memory function [Carrera et al., 1998; Russian et al., 2013] as
351 outlined below. The evolution of the hydraulic head in the mobile region (h_m) is described
352 by the non-local Dupuit equation [Russian et al., 2013]

$$S_m \frac{\partial h_m(x, t)}{\partial t} = T_m \frac{\partial^2 h_m(x, t)}{\partial x^2} + r'(t) + F_{im}(x, t), \quad (5)$$

353 where F_{im} is a source/sink term defined as

$$F_{im}(x, t) = S_{im} \frac{\partial}{\partial t} \int_0^t g(t - t') h_m(x, t) dt', \quad (6)$$

354 where $g(t)$ is the memory function defined below. For Dirichlet boundary conditions at
355 the outfall, the TF reads as [see Eq. C2 in Russian et al., 2013]

$$\text{TF}_{\text{DC}}(x_L, \omega) = \frac{1}{\omega^2 |S_m + S_{im}g(\omega)|^2} \left| 1 - \frac{\cosh \left[\sqrt{i\omega\tau_e(\omega)}(1 - x_L/L) \right]}{\cosh \left[\sqrt{i\omega\tau_e(\omega)} \right]} \right|^2, \quad (7)$$

where S_m and S_{im} are the storage coefficients of the mobile and immobile zones, respectively and $\tau_e(\omega) = L^2[S_m + S_{im}g(\omega)]/T_m$. The response time of the mobile domain is given by $\tau_m = L^2S_m/T_m$. The memory function $g(\omega)$ is defined in frequency domain as

$$g(\omega) = \frac{1}{\sqrt{i\omega\tau_{im}}} \tanh \sqrt{i\omega\tau_{im}}, \quad (8)$$

where τ_{im} is the relaxation time of the immobile zone.

The DC model is defined in terms of four parameters (transmissivity and storage coefficient of the mobile continuum, the storage coefficient and the relaxation time of the immobile continuum). Note that the formulations for the local and non-local DM are very similar; actually, (7) tends to (4) as $S_{im}g(\omega) \rightarrow 0$, which occurs when equilibrium is reached, this means for $\omega \ll \tau_{im}^{-1}$. To observe an impact of the immobile zone on the aquifer dynamics, the two relaxation time scales τ_m and τ_{im} must be clearly separated. This can be measured by the dimensionless 'activation number' A_C defined as

$$A_c = \left(\frac{S_m}{S_{im}} \right)^2. \quad (9)$$

If the activation number is $A_c < 1$ the system 'notices' an impact of the immobile zone. The smaller A_c , the larger the relevance of the non-local effects on the shape of TFs.

4.3. Derivation of Experimental Transfer Functions and Fitting Methodology

We computed experimental transfer functions (TF_{EXP}) from head fluctuation time series obtained from 136 boreholes existing in the site. We used continuous recordings from different time intervals, which presented a few gaps that were filled by linear interpolation.

365 The frequency of the measurements is constant for each piezometer, but varies from point
366 to point (*Supplementary Material*). We distinguished between two clusters of data, those
367 with measurement intervals of about 15-30 days, and those with measurement intervals
368 below or equal 1 day.

TF_{EXP} is calculated as the ratio between the power-spectral density of spatially variable
observed head fluctuations (PSD_h) and the power spectral density of the aquifer recharge
(PSD_r), as

$$TF_{EXP}(\omega) = \frac{PSD_h(\omega)}{PSD_r(\omega)}. \quad (10)$$

369 No specific recharge analysis has been performed on this site, thus we take it homoge-
370 neously distributed in the domain, which is a reasonable approximation for small basin.
371 To account for the high evaporation rates existing in the site (located in southern Spain),
372 recharge is estimated as half the total precipitation. Hourly rainfall time series were
373 collected at a meteorological station located in the basin. Runoff is assumed negligible.
374 PSD_h and PSD_r are computed using the MATLAB native function 'periodogram.m',
375 which adopts a nonparametric approach under the assumption of a wide-sense stationary
376 random process and using discrete Fourier transform (DFT). No regularization approach
377 was used to filter high-frequency signals, in order to minimize artificial spurious effects
378 that could bias the parameter estimations.

379 The relative distance of each borehole from the discharge location (x_L) is used as an
380 entry parameter in the models. Using a simple GIS-based calculation, x_L was obtained by
381 computing the minimum Cartesian distance from each borehole to the three main streams
382 identified in the catchment (Montesina, Morales, Arroyo 4). Of the 136 boreholes used in
383 this analysis,

- 384 • 16 boreholes are located at $x_L < 50\text{m}$,
- 385 • 34 boreholes are located at $x_L < 100\text{m}$,
- 386 • 76 boreholes are located at $x_L < 200\text{m}$,
- 387 • 116 boreholes are located at $x_L < 500\text{m}$.

388 As a working assumption, the catchment width is assumed to be constant for all bore-
389 holes and equal to $L = 1000\text{ m}$ (similar to maximum size of the three subcatchments). We
390 performed a sensitivity analysis using a specific L for each borehole, based on the size of
391 the individual catchment, but obtained no remarkable difference compare with the results
392 using a constant L (*Supplementary Material*). As such, the results shown hereafter refer
393 to a constant L value.

394 For each borehole, the TF for the DM, Eq. (4), and DC model, Eq. (7), were fitted to
395 experimental TFs using a non-linear least squares fit (MATLAB native function 'lsqcurve-
396 fit.m'). The procedure is based on the minimization of an objective function, imposing
397 a range of values and an initial estimation. The quality of the fitting exercise, measured
398 through the regression coefficient R^2 , was generally good (*Supplementary Material*).

4.4. Representative Example

399 Fig. 4 illustrates a representative example of experimental TFs . The top figure shows
400 the head fluctuation in one of the boreholes, with average reading interval of one day, and
401 in the middle the daily rainfall time series. At the bottom, the dotted gray line represents
402 the calculated experimental TFs for this borehole, which is overlapped by the two fitted
403 TF models (DC model in red and DM in blue). Two additional lines, scaling as $\text{TF} \propto \omega^{-1}$
404 and $\text{TF} \propto \omega^{-2}$, are shown for illustrative purposes. The plot is reported in double log
405 scales, which helps to infer the characteristic power-law distributions of the data.

406 Due to the noise, the characteristic shapes of theoretical TFs is only partially visible
407 from the experimental TFs. At low frequencies, TFs tend to be flat, reflecting long-
408 term hydraulic relationships (e.g., seasonal recharge processes). The lowest frequency
409 corresponds to $\omega = 1/365 \text{ d}^{-1}$. Around $\omega = 0.02$, the experimental data seem to decrease
410 at a rate $\omega^{-1} < TF < \omega^{-2}$, although the exact value is difficult to infer. For this specific
411 dataset, the maximum frequency is $\omega_N = 0.5 \text{ d}^{-1}$, where $\omega_N = 1/(2\Delta t)$ is the Nyquist
412 limit and $\Delta t = 1 \text{ d}$.

413 The two TF models displayed in Fig. 4 fit the dataset and help identifying the critical
414 features from these data, including the inflection points. The first inflection point is
415 found between low and mid frequencies ($\omega \approx 0.02$) and corresponds to the characteristic
416 response time of the mobile portion of the system (τ and τ_m , respectively for the DM
417 and DC models). We highlight that the scaling of this point is found at very similar
418 frequencies for both DC and DM, suggesting that the scaling of T, S and T_m, S_m may also
419 be similar.

420 Both models tend to scale as $TF = \omega^{-2}$ at higher frequencies, although DC seems to
421 reach this behavior earlier than DM. The reason is linked to the memory function term in
422 the DC model, which generates a second inflection point occurring at high frequencies. The
423 shape of the DC model is very similar to those predicted by the models by Molenat et al.
424 [1999] and Trinchero et al. [2011]. The former describes the effective discharge of a basin
425 as a combination of a fast hydrologic component such as lateral flow in the unsaturated
426 zone and/or overland flow, and slow flow such as groundwater aquifer discharge. The
427 latter combines the main groundwater flow to a more rapid component due to presence of
428 highly-connected preferential flow channels. In our datasets, this means that the model

429 is sensitive to the effects of short-term recharge processes and heterogeneity occurring at
430 El Cabril. The DM is not able to reproduce these short-term processes, being incapable
431 to reproduce this final scaling in the curve.

432 The behavior of experimental and theoretical curves shown in this specific example is
433 qualitatively similar to the general behavior of the entire analyzed dataset. However, the
434 exact scaling of the best-fitting models varies from borehole to borehole, and consequently
435 the resulting characteristic times (τ , τ_m and τ_{im}) and estimated parameters also signifi-
436 cantly fluctuate at the scale of the catchment. This reveals important aspects related to
437 scaling effects in estimated parameters, and the role of aquifer heterogeneity at El Cabril,
438 which is the key result for our work. These points are analyzed and discussed in the next
439 section.

440 It is ultimately highlighted that the high noise and the finite sampling frequencies may
441 bias the estimation of this second inflection point. This requires attention when inferring
442 behavior of the short-term recharge effects for a limited dataset, and can be seen as a
443 potential limitation of our analysis. A sensitivity analysis was run to quantify the impact
444 of the different sampling frequencies, comparing the spatial dependence of parameters
445 calculated exclusively from more complete and extended time series (having Nyquist limit
446 $N_f > 0.1$) against the results from the entire data set (which include extended and limited
447 time series. The results of this sensitivity analysis (*Supplementary Material*) seem to
448 suggest that the finite sampling frequencies may have only a moderate impact on the
449 estimated parameters and in particular on the scaling effects. Thus, the presence of high
450 noise and the finite sampling frequencies do not affect our main conclusion on parameter
451 estimation and scale effects.

5. Analysis and Discussion

5.1. Single Porosity Models – Scale Dependence of hydraulic parameters estimates

452 The resulting parameters obtained from TF-model fitting of the experimental dataset
453 is reported in Fig. 3. To emphasize the impact of scaling effect, we report the statistical
454 distributions of each parameter (in the form of cdfs), obtained from the ensemble of
455 boreholes located within specific x_L thresholds. This is done such that each cdf integrates
456 the impact of different heterogeneity scales on the hydraulic parameters.

457 We compare the estimates of T and S obtained with the TF of single-porosity DM for
458 different x_L and the ones obtained from slug and pumping tests.

459 Fig. 3 top illustrates the cdfs of T estimated with the different methods. For the
460 cdfs of T corresponding to boreholes found at $x_L < 100$ m, the mean values are smaller
461 than those estimated from the full population of pumping tests, while the degrees of
462 variability are comparable. As x_L increases, both the mean and variance of estimated
463 T also increase, reaching a maximum when the boreholes from the entire catchment are
464 considered ($x_L = L$). By accounting for x_L , the TF method generates scaling effects of
465 estimated T similar to those observed from traditional testing approaches. The distance
466 x_L can be interpreted as the support scale of the scale-dependent TF model. Indeed, when
467 the support scale is comparable with that of the pumping tests, the average T estimates
468 are similar, suggesting that TF may embed the same average amount of information
469 on aquifer transmissivity as traditional hydraulic testing approaches. This behavior is
470 explained using similar arguments as Le Borgne et al. [2006]: when x_L is small compared
471 to the scale of the catchment, the support scale of the TF is limited, a smaller number of
472 fractures is sampled and the resulting average T is low; when x_L is large, the scale of the

473 observation increases, the number of high-permeable zones increases and the average of
474 the estimated T values grows.

475 Note that, as the scale of observation increases, the relative variability of T also in-
476 creases, eventually becoming comparable with that of slug tests. This behavior, which
477 seems to contradict the evidences by Jimenez-Martinez et al. [2013], must be related to
478 the distribution of fractures in the aquifer. Similar to RQD for slug tests, the distribu-
479 tion of fractures in the aquifer is much broader than that of pumping tests. Hence, the
480 relative variability in T estimates cannot be directly compared with that corresponding
481 to pumping tests, since the effective support scale of TFs and pumping tests is biased by
482 a different geological background where the tests were performed.

483 For small support scales, the relative variability of T from slug-tests is much larger than
484 that from TFs. This is explained considering that slug tests provide a local estimation of T ,
485 which depends on the specific area of influence where the stress is locally applied. It is also
486 likely that slug tests tend to better sample the behavior of confined formations, which also
487 has a quicker elastic response than unconfined formations. Therefore, slug test estimates
488 become sensitive to the presence of small-scale heterogeneity in those formations. On the
489 other hand, TF provides an integrated vertically-averaged T value over the entire thickness
490 of the aquifer explored by the boreholes, in which multiple formations and fractured are
491 sampled. Most of the boreholes in this aquifer display a vertical distribution of RQD
492 similar to that of bh2 in Fig. 2 and each one samples multiple fractured zones, rather
493 than displaying a homogeneous distribution. In addition, it is also reminded that the
494 majority of head fluctuations is monitored through single-level piezometers and therefore
495 the response of the aquifer to surface recharge is also vertically averaged.

496 The behavior of T is consistent with the resulting behavior of storativity S illustrated
497 in the bottom panel of Fig. 3. For small x_L the the smallest estimated S values are
498 comparable with the largest values estimated from slug tests, whose relative variance is
499 larger than the one estimated by TF. As x_L grows, the relative variance of the TF data
500 is reduced and the estimated S scale at much larger average values than those from slug
501 and pumping tests. As the scale of observation grows, the system becomes effectively
502 equivalent to an unconfined formation. The weak dependency of the storativity with x_L
503 agrees with the hypothesis of vertical communication induced by the fracturing intensity of
504 the system. Fractures tend to facilitate communication between confined and unconfined
505 formations in the aquifer. The impact of fractures grows with the sampling scales, since
506 the observed fluctuation integrates a larger number of fractures at the observation points
507 departs from the discharge point.

508 We observed no significant changes in the distribution of T between $x_L < 500\text{m}$ and
509 the full data set. This may indicate that ergodic conditions are reached at 500 m. This
510 distance is shorter than the system's dimension but larger than the typical sampling scale
511 of pumping tests. There are important practical implications associated with this find-
512 ing. First, from a stochastic modeling perspective, it indicates that T may be simulated
513 as a stationary field for domains larger than 500m, while at shorter distance T can be
514 considered as a non-stationary field. Second, TF methods are less expensive than tra-
515 ditional hydraulic tests and therefore may gain importance to quickly estimate ergodic
516 scales. Indeed, TF statistics are obtained from a distribution of piezometers in the site
517 that do not need to be characterized using expensive hydraulic tests. Note that hydraulic

518 heads could be monitored over time using automatic data loggers, and thus be virtually
519 costs-free (excluding initial capital costs and maintenance operations).

520 We may conclude that a critical analysis of the distribution of geological features in
521 the aquifers and the actual position of observation points in the aquifer is fundamental
522 to correctly predict scaling effects in heterogeneous fractured formations. Most of the
523 apparent contradictions between our analysis and those reported by Jimenez-Martinez
524 et al. [2013] stem from both the different geological and lithological nature of the explored
525 aquifers and the use of a different interpretation model, specifically related to the explicit
526 spatial dependency of the observation point simulated by the DM.

5.2. Comparison Between Single Porosity and Dual-Continuum Formulations

527 We now compare the parameters estimated using the single domain DM against those
528 obtained from model fitting of the DC model described in Section 4.2. The estimated
529 hydraulic parameters are reported in Fig. 5, in the form of cdfs. Transmissivity and
530 storativity of the mobile domain (T_m , S_m) are plotted along with T and S from the single-
531 domain model, which are equivalent when the impact of immobile zone is not influencing
532 the results (i.e., $A_c < 1$). The results suggest that scale-dependent effects are still observed
533 for dual-domain estimates, although the estimates of both transmissivity and storativity
534 are different for the single and dual-domain TFs.

535 Regarding the transmissivity, we found a difference in the cdfs for $x_L < 100$ m depend-
536 ing on the adopted formulation. As the sampling scale increases, the difference in cdfs is
537 minimized. This result seems consistent with the potentially strong control of fractures,
538 connectivity and heterogeneity on the flow dynamics at short distances between observa-
539 tion and discharge point. The resulting 'anomalous behavior' of aquifer properties (which

540 can be effectively upscaled using non-local models, such as the dual continuum formula-
541 tion) occurs when the scale of observation is lower than or comparable to the characteristic
542 scale of heterogeneity. As the sampling scale increases, the aquifer becomes statistically
543 homogeneous and apparent non-local effects on flow dynamics disappear, which is similar
544 to the homogenization of solute transport in heterogeneous media as the scale sampled
545 by the solute increases [Zinn and Harvey, 2003; Dentz et al., 2004; Pedretti et al., 2014].

546 The estimates of storativity are much more sensitive to the model choice than trans-
547 missivity. At any scale, S_m is generally lower than S , while S_{im} tends to increase as
548 the sampling scale increases. It is observed, for instance, that approximately 50% of the
549 fitted boreholes found at $x_L < 100$ m display $S_{im} > 10^{-2}$, while the percentage increases
550 to about 75% when the entire catchment is explored. Consistently, $A_c \approx 1$ for about 40%
551 of the fitted boreholes at $x_L < 100$ m, while this percentage increases to more than 60%
552 when the entire catchment is explored. This suggests that the significance of the immobile
553 domain grows as the scale of domain increases.

554 The larger scale dependence of the storativity in the dual domain is due to the storage
555 capacity represented by the immobile domain through S_{im} . This component is not present
556 in single domain models, where all the storage capacity is lumped together into a single
557 S value. It is likely that, at larger sampling scales, the behavior of the El Cabil aquifer
558 superimposes a regional component, controlled by the mobile domain, and a local one,
559 controlled by preferential zones and small-scale fractures. Surface recharge controls the
560 vertical oscillation of the aquifer, relevant only in unconfined aquifers. Consistent with
561 what is discussed in the previous section for single domain models, the effective unsatu-
562 rated behavior of the system may be controlled by the number of fractures that generate

563 communication between confined and unconfined units. This number grows with the scale
564 of the observations. At short observation distances, the impact of vertical fractures is less
565 evident, explaining why the system is less sensitive to short-term recharge pulses.

566 We highlight, however, that at short observation distances the influence of both regional
567 recharge components and short-term local components may also somewhat overlap, re-
568 sulting in a mixed behavior on the TF which cannot be well fitted by non-local models.
569 While the sensitivity analysis seems to suggest that the impact of higher sampling fre-
570 quency does not qualitatively affect our conclusions (*Supplementary Material*), we specu-
571 late that a very refined time discretization (e.g., order of minutes) could result in different
572 tailing for experimental TFs at short-distance boreholes. This could be an indicator to
573 discriminate between single and dual continuum models at small x_L . No information is
574 nonetheless available so far to corroborate this hypothesis.

575 We therefore conclude that transmissivities are rather insensitive to single or dual-
576 domain interpretations of the data, since in both models water transmission is mainly
577 occurring through the mobile fracture continuum. The dual continuum model does not
578 consider transmission in the immobile matrix continuum, which provides a storage volume.
579 Thus, naturally, the storage capacities estimated for the single domain model and the
580 mobile storage capacity are quite different. Therefore, it is likely that the general response
581 of aquifer to recharge effects, which is controlled by the hydraulic diffusivity of the system,
582 could be affected by the presence of effective low-permeable zones which may affect the
583 transformation of recharge signals into head fluctuations. However, our analysis suggests
584 that care must be taken when inferring general conclusions based on dual continuum

585 models if the sampling frequency is limited, since the impact of immobile zones may not
586 be clearly observed.

6. Summary and Conclusions

587 We present an analysis of the scale dependence of hydraulic parameters (transmissivity
588 and storage capacity) based on a transfer function analysis. The transfer function ap-
589 proach considers the aquifer as linear filter, which can be characterized by comparison of
590 the power spectra of the input signals (aquifer recharge) and output signals (hydraulic
591 head fluctuation). Its dependence on frequency allows to infer information on the hy-
592 draulic aquifer properties based on a physical process model, which here is given by single
593 and dual-domain Dupuit aquifer models.

594 Unlike other approaches, we consider solutions to these models that explicitly account
595 for the distance between the location of the observation wells and the discharge boundary
596 (x_L). This allows for a scale-dependent interpretation of the response data from boreholes
597 at different locations, and thus to associate the estimated hydraulic parameters with a
598 given support or sampling scale of the TF solution. Thus, the estimation of transmissiv-
599 ity and storage capacity is based on analytical solutions for transfer functions that are
600 explicitly dependent on the distance to the recharge boundary.

601 We adopt the scale-dependent TF approach to analyze the El Cabil aquifer, which
602 is a well-characterized aquifer in Southern Spain. We first evaluate the data in view of
603 a scale dependence of transmissivity and storage capacity for the single domain model
604 and compare the results to estimates from slug and pumping test, which sample different
605 heterogeneity scales. We find that:

606 • The estimates for transmissivity show pronounced dependence on x_L and thus on the
607 sampling volume.

608 • The closest wells to the discharge point give results from the transmissivity distribu-
609 tion that are comparable in mean and variance to the ones obtained from pumping tests
610 because they consider similar support scales.

611 • The head data integrate heterogeneities both horizontally and vertically and have a
612 larger support volume than the slug tests, which give the smallest transmissivity estimates.
613 At increasing distance from the outfall, the TF support scales increase. Consequently,
614 both the mean and variability of the estimated transmissivity values increase.

615 • This scale effect in transmissivity is due to the non-stationary (maybe fractal) nature
616 of fracture length distributions, which implies that the probability to meet large connected
617 fractures increases with the sampling scale [see also Le Borgne et al., 2006]. Thus, hy-
618 draulic head data gives an inexpensive and efficient means to estimate local and global
619 hydraulic transmissivity.

620 • For the storage capacity, the scale effect is almost negligible, which indicates that
621 storage is due to vertical connectivity and short horizontal structures as implied by the
622 structural properties of the fractured aquifer.

623 We then tested the dual-porosity nature of the fractured aquifer by comparing estimates
624 from the single and dual-continuum aquifer models. We found that:

625 • The estimates for single-porosity transmissivity and mobile-domain transmissivity
626 were very similar because in the dual-domain model the immobile domain is not trans-
627 missive, but merely stores water.

628 • The estimates for the storage capacity in the single domain model and the mobile
629 storage capacity in the dual domain model are, as expected, very different.

630 • There is a scale dependence in the estimates for the immobile storage capacity which
631 allows determining the characteristic scales of the immobile domain. Thus, the transfer
632 function analysis based on a dual continuum model allows in principle to extract the dis-
633 tribution of immobile storativity and characteristic spatial scales of the immobile regions.

634 In conclusion, this analysis shows that the interpretation of hydraulic head data on
635 different scales through frequency analysis using transfer functions is an efficient and in-
636 expensive method for the estimation of hydraulic parameters. More than this, in principle
637 it allows to extract information on the heterogeneity scales and dual-domain nature of the
638 fractured medium.

Acknowledgments

639 XS acknowledges support from the ICREA Academia Project. MD acknowledges the sup-
640 port of the European Research Council (ERC) through the project MHetScale (617511).

641 The authors endorse AGU data policy. All the data and additional information used and
642 cited in this paper can be provided by the corresponding author (DP) at specific requests.

643 The authors acknowledge the useful suggestions provided by three anonymous reviewers,
644 who helped to improve the quality of our manuscript.

Figure 1. (a) geographical location of the El Cabril Site; (b) outcrop illustrating the intensity of rock fracturing in the site; (c) a representative drilling core box from one of the boreholes used to compute the RQD index; (d) the digital elevation model of the site, reporting the main surface hydrological patterns at the site (10^3 m scale); (e) the conceptualization of the groundwater dynamics at the site, including regional and local flow paths.

Figure 2. (a) Geological sketch of El Cabril (oriented NE-SW) and three representative stratigraphic columns with vertical distribution of the corresponding logs and the calculated RQD values; (b) Frequency histogram and cumulative density of RQD at the El Cabril site. Black colors refer to the distribution obtained from all existing data; red and blue refer to distributions obtained from the subsample of boreholes used for pumping and slug tests, respectively.

Figure 3. Cumulative distributions of (top panel) transmissivity (T , in m^2/d) and (bottom panel) storativity (S , dimensionless) obtained from (squares) slug tests, (triangles) pumping tests, and from fitting of experimental TFs with the Dupuit model associated to boreholes located at (grey circles) $x_L < 0.05$, (red solid) $x_L < 0.1$, (pink dashed) $x_L < 0.2$, (green dash-dotted) $x_L < 0.5$, and (blue dotted) data from the whole catchment.

Figure 4. From top to bottom, a representative example of the hydrograph from one selected borehole, the rainfall time series and the resulting experimental transfer function (TF_{EXP}), with the fitted models (DM=Single-porosity, scale-dependent Dupuit model; DC=Dual-Continuum version of the DM).

Figure 5. Cumulative distributions of (top left panel) transmissivities, (top right panel) activation number A_c , (bottom panels) storativity. Estimates from the Dupuit model (DM) are (red solid) for $x_L < 100$ m and (blue dash-dotted) for the all the catchment. Estimates from the non-local Dupuit model (DC) are (red dotted) for $x_L < 100$ m and (blue dashed) for the whole catchment.

References

- 645 B. Berkowitz, Characterizing flow and transport in fractured geological media: A review, *Ad-*
646 *vances in Water Resources* 25 (812) (2002) 861–884, ISSN 0309-1708, doi:10.1016/S0309-
647 1708(02)00042-8.
- 648 A. Zech, S. Arnold, C. Schneider, S. Attinger, Estimating parameters of aquifer heterogeneity
649 using pumping tests implications for field applications, *Advances in Water Resources* 83 (2015)
650 137–147, ISSN 0309-1708, doi:10.1016/j.advwatres.2015.05.021.
- 651 H. Zhou, J. J. Gómez-Hernández, L. Li, Inverse methods in hydrogeology: Evolution
652 and recent trends, *Advances in Water Resources* 63 (2014) 22–37, ISSN 0309-1708, doi:
653 10.1016/j.advwatres.2013.10.014.
- 654 V. Denic-Jukic, D. Jukic, Composite transfer functions for karst aquifers, *Journal of Hydrology*
655 274 (14) (2003) 80–94, ISSN 0022-1694, doi:10.1016/S0022-1694(02)00393-1.
- 656 Z. Liao, K. Osenbrück, O. A. Cirpka, Non-stationary nonparametric inference of river-to-
657 groundwater travel-time distributions, *Journal of Hydrology* 519, Part D (2014) 3386–3399,
658 ISSN 0022-1694, doi:10.1016/j.jhydrol.2014.09.084.
- 659 J.-L. Pinault, V. Plagnes, L. Aquilina, M. Bakalowicz, Inverse modeling of the hydrolog-
660 ical and the hydrochemical behavior of hydrosystems: Characterization of Karst System
661 Functioning, *Water Resources Research* 37 (8) (2001) 2191–2204, ISSN 1944-7973, doi:

662 10.1029/2001WR900018.

663 J. Molenat, P. Davy, C. Gascuelodoux, P. Durand, Study of three subsurface hydrologic systems
664 based on spectral and cross-spectral analysis of time series, *Journal of Hydrology* 222 (1-4)
665 (1999) 152-164, ISSN 00221694, doi:10.1016/S0022-1694(99)00107-9.

666 P. Trinchero, R. Beckie, X. Sanchez-Vila, C. Nichol, Assessing preferential flow through an
667 unsaturated waste rock pile using spectral analysis, *Water Resources Research* 47 (7) (2011)
668 n/a–n/a, ISSN 1944-7973, doi:10.1029/2010WR010163.

669 J. Jimenez-Martinez, L. Longuevergne, T. Le Borgne, P. Davy, A. Russian, O. Bour, Temporal
670 and spatial scaling of hydraulic response to recharge in fractured aquifers: Insights from a
671 frequency domain analysis, *Water Resources Research* 49 (5) (2013) 3007–3023, ISSN 1944-
672 7973, doi:10.1002/wrcr.20260.

673 L. W. Gelhar, Stochastic analysis of phreatic aquifers, *Water Resources Research* 10 (3) (1974)
674 539–545, ISSN 1944-7973, doi:10.1029/WR010i003p00539.

675 Y.-K. Zhang, K. Schilling, Temporal scaling of hydraulic head and river base flow and its im-
676 plication for groundwater recharge, *Water Resources Research* 40 (3) (2004) n/a–n/a, ISSN
677 1944-7973, doi:10.1029/2003WR002094.

678 K. E. Schilling, Y.-K. Zhang, Temporal Scaling of Groundwater Level Fluctuations Near
679 a Stream, *Ground Water* 50 (1) (2012) 59–67, ISSN 1745-6584, doi:10.1111/j.1745-
680 6584.2011.00804.x.

681 C. Barton, E. Larsen, Fractal Geometry of Two-Dimensional Fracture Networks at Yucca Moun-
682 tain, Southwestern Nevada, *Fundamentals of Rock Joints: Proceedings of the International*
683 *Symposium on Fundamentals of Rock Joints* (1985) 77–84.

684 E. Bonnet, O. Bour, N. E. Odling, P. Davy, I. Main, P. Cowie, B. Berkowitz, Scaling of fracture
685 systems in geological media, *Reviews of Geophysics* 39 (3) (2001) 347–383, ISSN 1944-9208,
686 doi:10.1029/1999RG000074.

687 Y.-K. Zhang, Z. Li, Temporal scaling of hydraulic head fluctuations: Nonstationary spectral
688 analyses and numerical simulations, *Water Resources Research* 41 (7) (2005) W07031, ISSN
689 1944-7973, doi:10.1029/2004WR003797.

690 M. A. Little, J. P. Bloomfield, Robust evidence for random fractal scaling of groundwater levels
691 in unconfined aquifers, *Journal of Hydrology* 393 (34) (2010) 362–369, ISSN 0022-1694, doi:
692 10.1016/j.jhydrol.2010.08.031.

693 W. F. Brace, Permeability of crystalline and argillaceous rocks, *International Journal of Rock*
694 *Mechanics and Mining Sciences & Geomechanics Abstracts* 17 (5) (1980) 241–251, ISSN 0148-
695 9062, doi:10.1016/0148-9062(80)90807-4.

696 W. F. Brace, Permeability of crystalline rocks: New in situ measurements, *Journal of*
697 *Geophysical Research: Solid Earth* 89 (B6) (1984) 4327–4330, ISSN 2156-2202, doi:
698 10.1029/JB089iB06p04327.

699 C. Clauser, Permeability of crystalline rocks, *Eos, Transactions American Geophysical Union*
700 73 (21) (1992) 233–238, ISSN 2324-9250, doi:10.1029/91EO00190.

701 J. Guimerá, L. Vives, J. Carrera, A discussion of scale effects on hydraulic conductivity at a
702 granitic site (El Berrocal, Spain), *Geophysical Research Letters* 22 (11) (1995) 1449–1452,
703 ISSN 1944-8007, doi:10.1029/95GL01493.

704 X. Sanchez-Vila, J. Carrera, J. P. Girardi, Scale effects in transmissivity, *Journal of Hydrology*
705 183 (12) (1996) 1–22, ISSN 0022-1694, doi:10.1016/S0022-1694(96)80031-X.

706 R. Beckie, Measurement Scale, Network Sampling Scale, and Groundwater Model Parameters,
707 Water Resources Research 32 (1) (1996) 65–76, ISSN 1944-7973, doi:10.1029/95WR02921.

708 J. Guimerá, J. Carrera, A comparison of hydraulic and transport parameters measured in low-
709 permeability fractured media, Journal of Contaminant Hydrology 41 (34) (2000) 261–281, ISSN
710 0169-7722, doi:10.1016/S0169-7722(99)00080-7.

711 D. Schulze-Makuch, P. Malik, The Scaling of Hydraulic Properties in Granitic Rocks, in: I. Sto-
712 ber, K. Bucher (Eds.), Hydrogeology of Crystalline Rocks, no. 34 in Water Science and Tech-
713 nology Library, Springer Netherlands, ISBN 978-90-481-5368-8 978-94-017-1816-5, 127–138,
714 2000.

715 J. Lai, L. Ren, Assessing the Size Dependency of Measured Hydraulic Conductivity Using Double-
716 Ring Infiltrimeters and Numerical Simulation, Soil Science Society of America Journal 71 (6)
717 (2007) 1667, ISSN 1435-0661, doi:10.2136/sssaj2006.0227.

718 T. Le Borgne, O. Bour, F. L. Paillet, J. P. Caudal, Assessment of preferential flow path
719 connectivity and hydraulic properties at single-borehole and cross-borehole scales in a
720 fractured aquifer, Journal of Hydrology 328 (12) (2006) 347–359, ISSN 0022-1694, doi:
721 10.1016/j.jhydrol.2005.12.029.

722 P. Meier, J. Carrera, X. Sanchez-Vila, An evaluation of Jacob’s Method for the interpretation
723 of pumping tests in heterogeneous formations, Water Resources Research 34 (5) (1998) 1011–
724 1025, ISSN 1944-7973, doi:10.1029/98WR00008.

725 X. Sanchez Vila, P. M. Meier, J. Carrera, Pumping tests in heterogeneous aquifers: An analytical
726 study of what can be obtained from their interpretation using Jacob’s Method, Water Resources
727 Research 35 (4) (1999) 943–952, ISSN 1944-7973, doi:10.1029/1999WR900007.

728 A. Russian, M. Dentz, T. Le Borgne, J. Carrera, J. Jimenez-Martinez, Temporal scaling of
729 groundwater discharge in dual and multicontinuum catchment models, *Water Resources Re-*
730 *search* 49 (12) (2013) 8552–8564, ISSN 1944-7973, doi:10.1002/2013WR014255.

731 A. F. Moench, Convergent Radial Dispersion in a Double-Porosity Aquifer with Fracture Skin:
732 Analytical Solution and Application to a Field Experiment in Fractured Chalk, *Water Re-*
733 *sources Research* 31 (8) (1995) 1823–1835, ISSN 1944-7973, doi:10.1029/95WR01275.

734 R. Haggerty, S. Gorelick, Multiple-rate mass transfer for modeling diffusion and surface reactions
735 in media with pore-scale heterogeneity, *Water Resour. Res.* 31 (10) (1995) 2383–2400, doi:
736 10.1029/95WR10583.

737 S. McKenna, L. Meigs, R. Haggerty, Tracer tests in a fractured dolomite. 3. Double-porosity,
738 multiple-rate mass transfer processes in convergent flow tracer tests., *Water Resour. Res.* 37 (5)
739 (2001) 1143–1154.

740 X. Sanchez-Vila, J. Carrera, On the striking similarity between the moments of breakthrough
741 curves for a heterogeneous medium and a homogeneous medium with a matrix diffusion term,
742 *Journal of hydrology* 294 (1-3) (2004) 164/175, doi:10.1016/j.jhydrol.2003.12.046.

743 BRGM, Operations de multitracages en ecoulement influence par pompage pour la determination
744 des parametres hydrodynamiques de laquifere superficiel du site d El Cabril (Espagne). Univ.
745 Politecnica de Catalunya, Barcelona, Tech. Rep., 1990.

746 J. Carrera, X. Sanchez-Vila, J. Samper, F. Elorza, J. Heredia, J. Carbonell, C. Bajos, Ra-
747 dioactive waste disposal on a highly heterogeneous fracture medium, 1, Conceptual models
748 of ground- water flow, in: *Proceedings of the International Association of Hydrogeologists,*
749 *XXIVth Congress, Hydrogeology of Hard Rocks*, edited by S. B. Banks and D. Banks, vol. 1,
750 pp. 190-202, Geol. Surv. of Norway, Trondheim, 1993.

- 751 X. Sanchez-Vila, J. Carrera, Directional effects on convergent flow tracer tests, *Mathematical*
752 *Geology* 29 (4) (1997) 551–569, doi:10.1007/BF02775086.
- 753 P. Trinchero, X. Sanchez-Vila, D. Fernandez-Garcia, Point-to-point connectivity, an abstract
754 concept or a key issue for risk assessment studies?, *Advances in Water Resources* 31 (12)
755 (2008) 1742–1753, ISSN 0309-1708, doi:10.1016/j.advwatres.2008.09.001.
- 756 D. Deere, Technical description of rock cores for engineering purposes, *Rock Mech. Rock Eng.*
757 1 (1) (1963) 16–22.
- 758 S. Priest, J. Hudson, Discontinuity spacings in rock, *Int J Rock Mech Min Sci* 13 (1976) 135–148.
- 759 Y. W. Tsang, C. F. Tsang, Flow channeling in a single fracture as a two-dimensional strongly
760 heterogeneous permeable medium, *Water Resources Research* 25 (9) (1989) 2076–2080, ISSN
761 1944-7973, doi:10.1029/WR025i009p02076.
- 762 G. Gustafson, A. Fransson, The use of the Pareto distribution for fracture transmissivity as-
763 sessment, *Hydrogeology Journal* 14 (1-2) (2005) 15–20, ISSN 1431-2174, 1435-0157, doi:
764 10.1007/s10040-005-0440-y.
- 765 J. Liu, D. Elsworth, B. H. Brady, Linking stress-dependent effective porosity and hydraulic
766 conductivity fields to RMR, *International Journal of Rock Mechanics and Mining Sciences*
767 36 (5) (1999) 581–596, ISSN 1365-1609, doi:10.1016/S0148-9062(99)00029-7.
- 768 S. S. Papadopoulos, J. D. Bredehoeft, H. H. Cooper, On the analysis of slug test data, *Water*
769 *Resources Research* 9 (4) (1973) 1087–1089, ISSN 1944-7973, doi:10.1029/WR009i004p01087.
- 770 J. Carbonell, J. Carrera, MARIAJ, FORTRAN code for the automatic interpretation of pumping
771 tests. User 's guide, E.T.S.I.Caminos. Barcelona, Spain: Universitat Politecnica de Catalunya.,
772 1992.
- 773 J. Bear, *Dynamics of fluids in porous media*, Elsevier, New York, 1972.

- 774 A. D. Erskine, A. Papaioannou, The use of aquifer response rate in the assessment of groundwater
775 resources, *Journal of Hydrology* 202 (1997) 373–391.
- 776 J. Carrera, X. Sanchez-Vila, I. Benet, A. Medina, G. Galarza, J. Guimera, On matrix diffusion:
777 formulations, solution methods and qualitative effects., *Hydrogeol. J.* 6 (1998) 178–190, doi:
778 10.1007/s100400050143.
- 779 B. Zinn, C. F. Harvey, When good statistical models of aquifer heterogeneity go bad: A compar-
780 ison of flow, dispersion and mass transfer in connected and multivariate Gaussian hydraulic
781 conductivity fields, *Water Resources Research* 39 (3) (2003) 1051, doi:10.1029/2001WR001146.
- 782 M. Dentz, A. Cortis, H. Scher, B. Berkowitz, Time behavior of solute transport in heterogeneous
783 media: transition from anomalous to normal transport, *Adv. Water Resour.* 27 (2) (2004)
784 155–173.
- 785 D. Pedretti, D. Fernandez-Garcia, X. Sanchez-Vila, D. Bolster, D. A. Benson, Apparent directional
786 mass-transfer capacity coefficients in three-dimensional anisotropic heterogeneous aquifers un-
787 der radial convergent transport, *Water Resources Research* 50 (2) (2014) 1205–1224, ISSN
788 1944-7973, doi:10.1002/2013WR014578.